JOURNAL OF AEROSPACE COMPUTING, INFORMATION, AND COMMUNICATION
Vol. 1, February 2004

General Method of the Automatic Generation of Onboard Triplet

Zheng Sheng,* Tian Jinwen,† and Liu Jian‡
Huazhong University of Science and Technology, Wuhan 430074, China

A novel general method of the automatic selection of onboard star triplet, namely triplet regression selection algorithm (TRSA), which based on a new dynamical label visual magnitude threshold (DLVMT) model, is presented. By defining the label visual magnitude and the direction of the star triplet, the star triplet distribution is analyzed. Using the DLVMT to filter the star triplet set, a new catalog with uniform distribution of the triplets over the celestial sphere can be obtained. The DLVMT distribution function has been attained via the support vector machines (SVM) regression method. With the proposed sampling method, computer experiments were carried out. The experiment results demonstrate that the triplet database obtained by the proposed algorithm has a couple of advantages, including fewer total numbers, smaller catalog size, and better distribution uniformity.

I. Introduction

The star trackers provided high-precision three-axis attitude information due to extremely accurate reference information as well as improvement of star sensing devices. Achievement of full autonomy of operation is the most important development trend of the modern star trackers,[1] with which their operation capability to cover most or even all mission phases can be widened and all attitude data required for control can be supplied. The development of full autonomy of operation is also in accordance with the requirements of saving power, mass, and volume, and of limiting complexity and redundancy of onboard systems. An autonomous star tracker can operate and manage independently different mission phase requirements without support from other spacecraft units except the star image. These phases include the start up routine to determine the rough localization of the observed region of the sky, and the normal tracking mode following the initial acquisition procedure to estimate the high-precision attitude of the spacecraft. These different specific features are usually attained via software procedures. To obtain full autonomous attitude estimation, the star tracker should perform a prompt identification of the viewed star field by comparing observed star features and star characteristics stored in its onboard catalog. Once a correct match is made, there are reliable methods for generating good attitude estimation.

Recently, many star pattern recognition (SPR) algorithms to generate a best match between the measured star pattern in the FOV and the subimage of the onboard catalog have been proposed. According to their respective identification approaches in the FOV, these algorithms can be divided into three classes. The first class of algorithms is the inter-star pair that has angular separation-based matching methods, in which the stars are treated as vertexes in a graph whose edges correspond to the angular separation between neighboring stars that could possibly share the same sensor FOV, such as those from Refs. 1 and 2. The grid algorithms, such as those from Refs. 3 and 4 belong to the second class of algorithms, in which the well-defined pattern determined by the surrounding star field has been associated with every star. The third class of algorithms is the developing neural networks-based recognition algorithms,[5] in which the star images of the FOV are treated as patterns that can be recognized directly.

Because the neural network structure itself contains the information about the star feature vectors, the precompiled star feature database is not necessary to the neural networks-based star identification strategies. Except

---

Received 20 May 2003; accepted for publication 23 October 2003.
*Ph.D. candidate. State Education Commission Key Laboratory for Image Processing and Intelligent Control, Inst. for Pattern Recognition and Artificial Intelligence; Zhengsheng6511@sohu.com, zsh@ctgu.cn. Student Member AIAA.
†Professor and Ph.D. supervisor. Inst. for Pattern Recognition and Artificial Intelligence; jwtian@mail.hust.edu.cn.
‡Professor and Ph.D. supervisor. Inst. for Pattern Recognition and Artificial Intelligence; bliu@mail.hust.edu.cn.

the third class algorithms, all algorithms make use of an onboard catalog that consists of a set of known star characteristics, usually composed of the right ascension and declination of the stars or direction cosines and their visual magnitudes, and precalculated information such as the inter-star angular separations that aid in pairing sensor stars to the appropriate catalog stars and also operate in tracking mode or start up routine. To reduce the possible mission threatening effects that result from the loss of reliable guidance control information due to temporary power failure or system malfunctioning or initialization, the star tracker's capability of capture of the attitude when lost in space is significant to a deep space probe. As stars are typically seen in all possible directions and fixed sensors are used to image a portion of sky, the stars extracted from the FOV can be used to establish a correspondence to a portion of sky in an onboard star catalog. As long as the system is able to match at least two of the sensor stars, there is sufficient information to reliably determine the attitude of the spacecraft with respect to the reference frame of the catalog. Because the current attitude of the spacecraft can be used to estimate the next rough localization of the FOV, the star identification in tracking mode is relatively easy because the searching range is restricted in the neighboring area of the rough orientation.

It should be noted that many brightness-independent SPR algorithms have been developed, such as those in Ref. 1, because the probability of the correct star field recognition is considerably influenced by star brightness estimation errors when brightness-based algorithms are used. This uncertainty is mainly due to the measurement inaccuracy of charge-coupled-devices (CCD)-based star sensors. These brightness-independent SPR algorithms can take advantage of the highly accurate, and widely implemented and tested image processing techniques available for the computation of the apparent position of viewed stars.

Typically the data structures in the onboard database employed by the first class SPR strategy includes lists of star pair distances or triplets from the catalog that are used to aid in constructing a subimage similar to the sensor graph. As sensor accuracy is not perfect, there are often numerous pairs or triplets that match a given sensor pair or triplet. Because more parts of the sensor graph are matched, many of the database subgraphs can be eliminated until only a single subgraph remains.

The triplet-matching technique is one of the simplest and most frequently used SPR algorithms. The star triplet is the basic structure in the matching procedure. It is easy and the preferred method to use when the number of stars viewed inside the FOV is fewer. It is worth noting that the performance of the matching procedure is considerably influenced by star triplet selection and the triplet database organization.

First, the angular separation between triplet vertexes should be within a specified range, because the angular separation between pairs of viewed stars always has the measurement uncertainty, which introduces errors in the computation of star positions through the subpixel technique by centroiding multiple-pixel images such as using pixel intensity as weight. For a specified tracker, the system measurement error can be estimated. We assume that the maximum measurement error of the angular separation is 0.01 deg. To have an angular measurement error that is always less than 1% for example, the lower bound of the angular separation should be set to at least 1 deg. The upper bound of the angular separation should be set at 7/8 of the size of a specified FOV, for example, for an 8 x 8 deg squared FOV, 6.99 deg is rational, to minimize the occurrence that one or more stars of a triplet could lie outside a squared FOV.

Second, the number of star triplets should be minimized and the distributing uniformity should be maximized for the performance and reliability of the triangle-based SPR algorithm. Stars in the guide star catalog can construct many triplets within the specified edge distance. If we add all these triplets to the onboard catalog, it can cause some undesirable effects on the star tracker performance. Because the guide stars obtained by the general selection algorithms did not consider the angular separations of the star pairs and the number of triplets do not evenly distribute over the celestial sphere, the onboard triplet database will have the excessive number of triplets in some boresight directions leading to ambiguity in the SPR and the lack of triplets in other directions leading to discontinuity of the star tracker operation and the reduction of the reliability of the attitude acquisition. At the same time, the redundant number of star triplets of the onboard catalog led to an increase in storage space and searching time and reduced the efficiency of the matching procedure. Therefore the star triplet database containing sufficient star triplets should be filtered once more so that the mission triplet database has the least number of star triplets and maximal distribution uniformity over the celestial sphere. This should be done while ensuring the least n number of star triplets with the specified range of angular separation of star pairs to be measured inside the specified FOV in any boresight directions.

To obtain a well-defined guide star catalog, some approaches have been proposed. In Ref. 6 a heuristic approach is proposed to obtain the guide start set consisting of the near minimum number of stars, ensuring that at least n (e.g. $n = 3$) guide stars are measured inside a specified FOV in every possible boresight direction, while the stars selected for the guide star catalog are brighter than some sensor-dictated magnitude threshold. The authors in Ref. 7 introduced an equal area mapping method, in which all candidate stars are projected from the unit sphere to a rectangle and divided with orthogonal grids of equal areas so that the redundant stars can be properly eliminated. The weighted visual magnitude method was proposed in Ref. 8, in which the weighted magnitude is used to control the selection of the guide star. However, these methods do not consider the angular separation of the star pairs. They cannot ensure that the angular separation of guide star triplets generated from those guide star sets are within a specified distance range and that the star triplets will distribute evenly. Using the existing procedure, the authors of Ref. 1 generate a triplet for each element of the onboard catalog by searching two other stars in a spherical corona centered in the first one. Lower and upper spreads of the corona are set taking into account the angular measurement accuracy of the sensor, the FOV size, and the onboard star catalog distribution density. Whereas, this approach is based mainly on the enumerating method and lacks the statistical theory support, it is not efficient in effectively avoiding holes and reducing the number of guide star triplets while ensuring that the guide star triplets distribute evenly. According to the traditional statistical theory, the optimist result can only be attained when the number of the sampling data runs to infinitude. It is an extremely high dimensional and nonlinear optimization problem. It is impossible to solve the problem by the usual approaches in the finite time.

The development of the statistical learning theory (SLT) and the SVM provide us a new approach with clear connections to the underlying statistical learning.[9,10] SLT is a statistical theory which focuses mainly on the statistical principles with small samples. Compared with the traditional statistical theory, the purpose of SLT is to obtain the optimist solution with the limited samples and not attain the best result with an infinite number of training data. SVM is based on principles of structural risk minimization and SLT. One nice feature about using SVM is that the computational capabilities of the machine are controlled by using the kernel functions. First, in the dual space the computational complexity of the problem depends on the number of observation examples and not the dimension of the input feature space. Second, SVM allows the expansion of the information contained in an initial database as a linear combination of a subset of the data in the training set (support vectors). This expansion has the capability of generalization over the complete star triplet set, even that not included in the training triplet set. Currently, SVM has reached a high level of maturity, with algorithms that are simple, easy to implement, and fast. They perform well and are successfully applied in pattern recognition and object detection,[10] function estimation,[11] and gene analysis[12] etc. Therefore the advantage of this property of the SVM is used in our method to generate the optimal guide star triplet catalog.

This paper addresses the generation of an optimal onboard triplet database with the minimal number and the maximal distribution uniformity of the star triangle. The starting point is the definition of the label visual magnitude and the direction of the star triangle. With the direction and visual magnitude, it is observed that the star triplets in the specified squared FOV in the celestial sphere are not distributed evenly. In addition the label visual magnitude threshold filtering the triplet set, that is generated from the basic guide star catalog, to obtain a mission set with even distribution of triplets varies at different boresight directions. It is reasonable to assume that using the DLVMT in the triplet filtering method can generate a desirable mission triplet database. In this way, the optimal selection problem of the star triplet has been transformed into obtaining the optimal DLVMT function, which can be solved by the regression theory and method. However, the DLVMT function is a high dimensional and nonlinear regression problem, which can be solved by a new approach that combines statistical learning theory and SVM. This is the novel method we proposed, namely SVM-based TRSA for the onboard star triplet database, which is derived from the DLVMT model.

To verify the proposed algorithm, different sets of star triplets are generated for the star tracker with an 8 x 8 deg squared FOV. The statistics of the number of triplets measured inside the 8 x 8 deg squared FOV using the star triplet sets identified by the TRSA, tested in 10,000 random boresight directions, are obtained. The experiment results demonstrate that the triplet database obtained by the SVM-based TRSA has a couple of advantages, including fewer total numbers, smaller database size, and better distribution uniformity.

Following is an overview of this paper. A description of the direction and label visual magnitude (LVM) of the star triplet will be discussed in Sec. II. The distribution of the direction and LVM of the star triplet will be described in Sec. III. The SVM and the regression are briefly described in Sec. IV. The approaches described in Secs. III and IV are incorporated into the selection of the onboard guide star triplet. The TRSA algorithm is described in Sec. V. The experiment results and discussions are provided in Sec. VI and conclusions are drawn in Sec. VII.

II. Direction and LVM of the Star Triplet

The general approaches of the selection of the guide star can generate the guide star catalog with fewer total numbers and better uniformity distribution than the star set obtained by using the magnitude filtering method (MFM). However they cannot ensure that the angular separation of the star pairs in the catalog can be constrained in a specified distance range and that star triplets generated from the star catalog will distribute uniformly. To generate the onboard triplet database consisting of a near minimum number of triplets, and the triplets in the database uniformly distribute over the celestial sphere, while the angular separation of the star pairs in the selected triplets is constrained in the specified range, the additional star triplets filtering procedure should be executed. To obtain the desirable mission triplet catalog, it is important to understand the distribution of the star triplet over the celestial sphere. The first is to define the features that present the direction and visual magnitude of the star triplet. Many parameters of the triangle can be available, such as the area, perimeter, center of mass etc. Here, the focus is on the center of the mass of the triangle. A center of mass of a plane can be defined as follows

$$\begin{aligned} x_0 &= M_{10} / M_{00} \\ y_0 &= M_{01} / M_{00} \end{aligned} \tag{1}$$

where M is the torque of the plane, if the mass function f (x,y) of the plane is discrete, then

$$\begin{aligned} M_{00} &= \sum_x \sum_y f(x,y) \\ M_{01} &= \sum_x \sum_y xf(x,y) \\ M_{10} &= \sum_x \sum_y yf(x,y) \end{aligned} \tag{2}$$

Figure 1 shows a star triplet in the inertial frame of the celestial sphere. Assume the mass of the triplet is 0 except the three vertexes are 1. Then the center of the star triplet (1) can be rewritten as follows

$$\begin{aligned} x_0 &= \frac{1}{3} \sum_{i=1}^{3} x_i \\ y_0 &= \frac{1}{3} \sum_{i=1}^{3} y_i \end{aligned} \tag{3}$$

where $(x,y)\left(x \in [0,2\pi], y \in [-\pi/2,+\pi/2]\right)$ is the direction of the stars of the triplet, that is the right ascensions and declinations. In this way, the center of the mass of the star triplet may be used to describe the direction of the triplet in the celestial sphere. Therefore the star triplets are distributed over the sky at different directions. To decide whether the triplet should be measured inside the specified FOV, the three edges should be considered because the direction inside the specified FOV cannot ensure that the three vertexes of the star triplet are measured inside the FOV. If we use the direction and the three edges as the star triplet features, the triplet can be fixed uniquely in the celestial sphere.

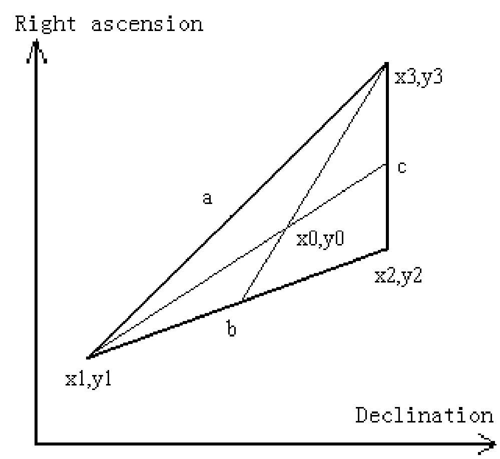

Fig. 1 Star triplet.

Because the bright stars are imaged with higher signal noise ratio (SNR) than the faint ones this results in a more accurate estimate of their apparent position. Hence, they enjoy a preferential right in selection when compiling the onboard start triplet database, therefore the maximum visual magnitude of three stars may present the visual magnitude of the star triplet, called the label visual magnitude (LVM). The smaller the LVM, the higher the SNR when the triplets are imaged. It should be preferred to be selected as the guide star triplet, and then added into the onboard triplet database.

Based on the direction and the LVM, the star triplet can be regarded as the basic star, then the method of the analyzing the distribution of the star can be used to analyze the distribution of the star triplet in the celestial sphere.

III. Triple's Distribution

To obtain the desirable onboard triplet database, it is important to observe the distribution of the star triplet over the celestial sphere. To analyze the distribution, the available star triplet set should be constructed first. A triplet is generated for each element of the onboard guide star catalog by searching two other stars in a spherical corona centered in the first one. Lower and upper spreads of the corona are set taking account of the angular measurement accuracy of the sensor and the FOV size. Here the angular separation of star pairs is restricted in the range of 1 deg to 6 deg. Figure 2 shows the distribution of the number of measurements inside the 8 x 8 deg squared FOV using star triplet sets generated from the 7817 stars in the catalog with 28,7540 triplets at different boresight directions sampled evenly with 0.1 radian intervals.

Obviously, the triplets of the numbers measured inside the FOV are not evenly distributed. However it is shown that the general selection approaches of the guide star cannot ensure the star triplet distribute evenly. Table 1 shows the statistics of a number of triplet/star measurements (M) in different guide star triplet/star sets for 8 x 8 deg squared FOV tested in 10,000 random boresight directions. It is shown that the basic guide star set has no holes and approximately least number of stars, as well as very good star distribution uniformity, e.g. 7,817 stars set. Whereas, the average number, the difference between the maximum and the minimum number and the standard deviation of the number of measurements of star triplets is very large, the range of the number of measurements is broader and the triplet distribution is not uniform. Therefore, a general selection algorithm of the guide star may generate a good guide star set, but cannot ensure that the triplet catalog will be good also. The significance of the optimal selection of the onboard triplet is clearly manifested.

The total number of triplets increases swiftly with the increase of the number of stars of the guide star set, e.g. about 55,189 star triplets are found for 4802 star sets while 287,540 triplets are found for 7817 star sets for a specified range of the angular separation. With the increase of the range of the angular separation of the star pair for a guide star set, the number of star triplets also increases very fast, e.g. for 7817 star sets, when the range is 1-6 deg the number of the triplets is 287,540; when the range is 1-6.99 deg, the number of triplets is up to 561,435. The statistics on the number of triplets shows that the usual guide selection algorithm cannot eliminate the holes of the star triplet at some range of the angular separation, e.g. 4,802 stars set about 1.22% of the holes of the triplet within the specified range 1-6 deg of angular separation is exist. Other problems happen when one tries to reduce the holes by increasing the range of the angular separation or increase the number of guide stars so as to increase the reliability of the initial attitude acquisition, which makes the number of guide star triplets increase largely. Storage

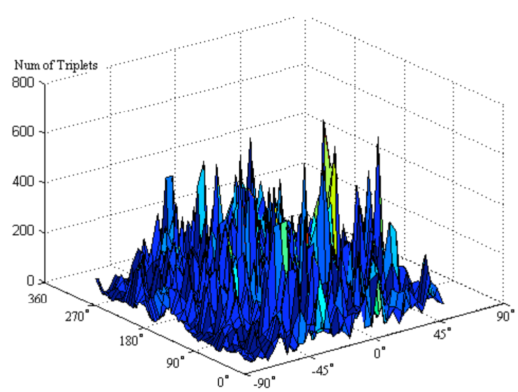

Fig. 2 Distribution of direction of the star triplets measured inside 8 x 8 deg FOV with 287,540 triplet sets at different boresight directions sampled evenly with 0.1 radian intervals.

Table 1 Statistics of number of triplet/star measurements(M) in different triplet/star sets for 8 x 8 deg squared FOV tested in 10,000 random boresight directions

Angular separation	Mean	Min	Max	Std	M <1	<2	<3	<4	<5	5-12	>12	Guide Stars	Triplets
1° - 6°	22.24	0	168	17.86	122	291	600	776	1161	2235	6604	802	55189
1° - 6°	116.73	0	1013	98.26	4	8	14	17	25	155	9820	7817	287540
1° -6.99°	170.56	0	1337	140.90	1	1	7	10	15	48	9937	7817	561435
	7.31*	1	14	1.77	0	1	8	100	505	9477	18	4802	7.9**
	11.92*	4	25	2.79	0	0	0	0	4	6177	3819	7817	7.5**

[a]denotes the statistic of the number of star measurements
[b]7.9 and 7.5 denote the highest visual magnitude in the star set

and searching time increase as well. If the magnitude information is excluded for the SPR and measurement errors exist, the SPR will require at least two matching star triplet pairs to avoid mismatches, one will need at least 7817 star sets to avoid triplet holes and mismatches for squared 8 x 8 deg sampling FOV, about 0.04% for 287,540 triplets set, and 0.01% for 561,435 triplets set. Such a number of guide star triplets can cause a storage problem and pattern match ambiguities, as well as slow SPR. Obviously, the star triplets are unevenly distributed in the celestial sphere, and the second selection of the guide star triplets is necessary and very important to the SPR.

Figure 3 shows the distribution of the LVMT with equal star triplets density for squared 8 x 8 deg FOV while four star triplets can be measured at different boresight directions sampled with even 0.1 radiant intervals, by using the 287,540 triplets set. It is shown that the LVMT varies at different boresight directions, and the distribution of the LVMT is actually a two-dimensional bend surface in the celestial sphere. Motivated by the thought of the MFM of the guide star, we developed a novel star TRSA, by introducing a new DLVMT model. With the DLVMT model, the star triplet sets can be filtered into two classes: onboard guide star triplets and nonmission guide star triplets. Where star triplets with LVM smaller than or equal to the corresponding LVMT in star triplet sets are chosen as onboard guide star triplets, while star triplets with LVM larger than the corresponding LVMT in a star triplet catalog are not chosen as onboard guide star triplets, the DLVMT is used as the star triplets filtering threshold. In this way, the selection problem of the onboard guide star triplet is converted to obtain the DLVMT function, which can be solved by the regression theory and method. Given the specified size of the FOV and the least number of star triplets measured inside the FOV, using the sampling method in the celestial sphere, LVMT at different boresight directions can be attained. Then the problem to select the onboard guide star triplet is transformed into the regression problem of the LVMT, where the input is the direction, and the output is optimal LVMT. We can describe the regression problem as follows. Given the training data set $\{(x_i, y_i)\}_{i=1}^{N}, x_i \in R^n, y_i \in R$, where x is the direction, y is the LVMT to obtain the optimal DLVMT regression function. Using the DLVMT, the desirable onboard triplet database consisting of near least number of triplets, in which the triplets uniformly distribute over the celestial sphere,

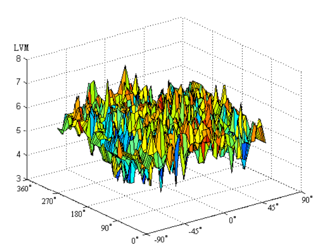

Fig. 3 Distribution of the LVMT with equal star triplets density for squared 8 x 8 deg FOV while four star triplets are measured at different boresight directions sampled with evenly 0.1 radiant intervals.

ensuring that at least n (e.g., n = 2) triplets are measured inside a specified FOV in every possible boresight direction can be attained. From the Fig. 3, it is obviously found that the problem to obtain the optimal DLVMT function is not a simple one-dimensional and linear problem but a two-dimensional and nonlinear problem. It is difficult to solve this problem with usual approaches within finite time. The development of the SLT and the SVM provide us a new approach with clear connections to the underlying statistical learning.

IV. Brief Introduction of the SVM and Regression

A. SVM Classifier

SVM, as a new neural network developed in recent years, is based on principles of structural risk minimization and SLT.[9-11] Given training data set $\{(x_i, y_i)\}_{i=1}^{N}, x_i \in R^n, y_i \in \{-1,+1\}$, the SVM classifier is virtually to solve the quadratic programming (QP) problem in the dual space. Although the SVM is developed on the linear binary classification, it can be easily extended to the high-dimensional and nonlinear classification problem. If the surface separating the two classes is nonlinear, the data points can be transformed to another high-dimensional feature space where the problem is linearly separable.

Let the transformation be $\varphi(\cdot)$. The Lagrangian function in the high-dimensional feature space is

$$Q(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j \varphi(x_i) \cdot \varphi(j) \tag{4}$$

Suppose, $K(x_i, x_j) = \varphi(x_i) \bullet \varphi(j)$. That is, the dot product in that high-dimensional feature space defines a kernel function of the input space. Therefore, it is not necessary to be explicit about the transformation as long as it is known that the kernel function corresponds to a dot product in some high dimensional feature space.[11,13]

There are many kernel functions that can be used, for example, the Gaussian radial basis function (RBF) kernel, $K(x, x_i) = \exp\left\{-|x - x_i|^2 / \sigma^2\right\}$, and the polynomial kernel, $K(x_i, x_j) = [(x \bullet x_i) + 1]^q$. The characterization of a kernel function $K(x_i, x_j)$ is done by means of the Mercer theorem.[10] With a suitable kernel in the feature space, SVM can separate the data that in the original input space was inseparable. This property means that we can obtain nonlinear algorithms by using proven methods to handle linearly separable data set.[13]

The SVM can also be extended to allow for imperfect separation by introducing non-negative slack variables to the hyper-plane and adding to the objective function a penalizing term. Then, using Lagrange multipliers and Karush Kuhn Tucker (KKT) conditions the decision function can be obtained and written as[13]

$$f(x) = \mathrm{sgn}\left(\sum \alpha_i^* y_i K(x_i, x) + b^* \right) \tag{5}$$

B. SVM and Regression

The regression problem can be considered a classification issue under the precision, ε.[11] The SVM for regression using a kernel function mapping the learning data (nonlinearly) into a higher-dimensional feature space where the computational power of the linear learning machine is increased, and the ε-insensitive loss function is formulated as,

$$\begin{aligned} \min \quad & \frac{1}{2}\|w\|^2 + \frac{1}{N} \sum_{i=1}^{N} \left(\xi_i + \hat{\xi}_i\right) \\ s.t. \quad & \left(w \bullet x_i + b\right) - y_i \le \varepsilon + \xi_i, i = 1, \cdots, N \\ & y_i - \left(w \bullet x_i + b\right) \le \varepsilon + \hat{\xi}_i, i = 1, \cdots, N \\ & \xi_i, \hat{\xi}_i \ge 0, i = 1, \cdots, N \end{aligned} \tag{6}$$

for a given learning data set $\{(x_i, y_i)\}_{i=1}^{N}, x_i \in R^n, y_i \in R$.

A standard optimization technique to solve QP problems is by solving the dual problem through the introduction of Lagrange multipliers. For each constraint in Eq. (6) there is a corresponding Lagrange Multiplier. Let the Lagrange multipliers for the problem in Eq. (6), $\alpha_i, \hat{\alpha}_i$ correspond with the constraints in Eq. (6) defined by the data point respectively. The combined weight of each data point x_i is determined by $\hat{\alpha}_i - \alpha_i$. The approximating function is given in terms of weights, the kernel and the bias,

$$f(x) = \left(w \bullet \phi\left(x\right)\right) + b = \sum_{i=1}^{n} (\hat{\alpha}_i - \alpha_i) K\left(x_i, x\right) + b \tag{7}$$

In our problem, the regression function is the DLVMT function.

V. Triplet Regression Selection Algorithm

Given the star triplet sets, with the introduction of a DLVMT model, the onboard triplets selection problem has been transformed into a regression problem, and aims to obtain the optimal DLVMT function which can approximate the optimal LVMT at any boresight direction to filter the star triplet set. The optimal DLVMT function can be easily found by the SVM. The procedure of the SVM-based TRSA algorithm is listed as follows.

Procedure 1: Prepare the star triplet sets needed to obtain the learning data set for the training SVM predictor.

1) Give guide star sets including sufficient stars needed in the generation of star triplets, which generated by the usual guide star selection strategies, e.g. 7,817 stars set.

2) Give the range of angular separation of the star pairs, e.g. 1-6 deg.

3) Calculate the available star triplets from the specified guide star sets while the length of the edges is within the specified range. Every triplet record is composed of the three edges (the angular separation) sorted on the length declination and the maximum visual magnitude and the directions (right ascensions and declinations) of the three stars.

4) Compute the direction of the star triplets and assign the maximum visual magnitude of the three stars as the LVM of the triplet.

5) Formulate one guide star triplet. The numbers, the edges, direction and LVM of the triplet are added into the guide star triplet set.

6) Repeat Steps 3-5 until the generation of the star triplet set has been completed.

Procedure 2: Prepare the learning data set needed to train the SVM predictor by sampling at different directions in the celestial sphere with the prepared basic triplet set.

1) Give the size of the specified squared FOV (e.g. 4 x 4 deg), the least number of triplets measured inside the FOV (e.g. 2 deg) and the initial triplet set with sufficient number of triplets prepared during Procedure 1.

2) Assume the triplets are evenly distributed in the specified FOV, sampling at different boresight directions with evenly sampling intervals (e.g. FOV/2).

3) Compute and count the number of directions of star triplets measured inside the specified FOV.

4) Sort the triplet directions measured inside the specified FOV on the triplet's LVM.

5) According to the least number of triplets measured inside the specified FOV, determine the LVM at the sampling direction. e.g. if the least number is 2, the LVM of the second triplets sorted on the LVM inside the FOV will be labeled as LVMT at the sampling direction. Here, star brightness has been implicitly taken into account since the bright stars are imaged with higher SNR than faint ones, which result in a more accurate estimate of their apparent position.

6) Formulate one training data. The feature x is the sampling direction (right ascensions and declinations), The y is the LVMT at the sampling orientation.

7) At another sampling direction, repeat Steps 2-6, until all orientations have been sampled in celestial sphere, obtain the training data set.

Given the different sizes of the specified squared FOV, the least number of triplets measured inside the FOV, the sampling intervals, and the initial triplet set, we can obtain different learning data sets fitting to different tasks.

Procedure 3: Choose the kernel type (e.g. RBF-kernel) and optimal hyper-parameters of the SVM, train the SVM predictor with the training data set and obtain the optimal DLVMT regression function.

Procedure 4: Estimate the LVMT by SVM predictor, filter the triplet set, and generate the onboard star triplet catalog.

1) Choose a triplet from the prepared star triplet set, the SVM predictor's input is the chosen triplet's direction (right ascensions and declinations).

2) Determine the triplet's class type using the regression estimate of LVMT by SVM predictor. If the triplet's LVM is smaller than or equal to the SVM predictor's estimate, then the triplet is added into the onboard triplet database as the guide star triplet; or not.

3) Choose another triplet in the triplet data set. Repeat Steps 1-2 until all triplets in the triplet set have been chosen and checked. Finally, the onboard guide star triplet catalog is generated.

Procedure 5: Check the newly created onboard star triplet catalog's effectiveness and perfection. If the catalog meets the design requirements, the TRSA is done, otherwise choose another group of hyper-parameters of the SVM predictor and return to Procedure 2.

VI. Results and Discussions

The SAO catalog[14] was selected in this paper, and the 7817 stars set is used as the basic guide star set. To demonstrate the SVM-based TRSA, different sets of guide star triplets are generated from 287,540 triplet sets for the star tracker with the 8 x 8 deg squared FOV using different sampling FOV size 4.5 deg and 5 deg respectively, where the number of star triplets measured inside the sampling FOV is four and the sampling intervals is half of the sampling FOV. The OSU-SVM[15] algorithm is used to train the SVM and filter the basic triplet set. Table 2 shows the statistics of the number of triplets measured inside the 8 x 8 deg squared FOV using the different star triplet sets, tested in 10,000 random boresight directions.

First, consider the influence of the sampling method and the parameters of the SVM. It is obviously shown that the star triplet sets are influenced by the sampling procedure and hyper-parameters of the SVM. When the hyper-parameter of Ga is constant, e.g. 150, the different sampling method leads to different results, e.g. with 4-5, the number of triplets is 93,912 with 7421 stars; when 4-4.5, the results are 142,174 with 7720 stars. On the other hand, at the same sampling method, different hyper-parameter of Ga also led to different results. If the method is 4-4.5, when Ga=150, the number of triplets is 142,174 with 7720 stars, while the Ga=200, the result is 139,815 with 7744 stars. Therefore, the optimist guide star triplet catalog can be obtained several times.

Second, compared the results with the Table 1, the total number of star triplets selected by the TRSA is obviously less than the number shown in Table 1, almost all of the star triplets are less than half of the original triplet's number, e.g. the total number is reduced from the 287,540 to the 142,174 of the 4-4.5 when Ga = 150, while the number of stars is also reduced from 7817 to 7720. Although the number of the triplets and the stars is reduced largely, but the distribution is better, the STD is reduced from the 100.12 to 38.06, which means the distribution is much more even. For the SPR, the minimum number of star triplets should be greater than or equal to 2. There are only 0.11% of boresight directions located in the holes where the number of triplets measured in the FOV is smaller than 2. Whereas, when the number of triplets is 146,461 while the number of stars is 7680, the proportion of orientations located in the holes where the number of viewed triplets is smaller than 2 is almost equal to the original proportion, only 0.08%. At the same time, the average number and maximum number of the triplets is almost one third of the 287,540 triplets set.

Table 2 Statistics of the number of triplets measurement (M) inside the 8 x 8 deg squared FOV using the different star triplet sets, tested in 10,000 random boresight directions.

						M								
Method	Ga	Mean	Min	Max	Std	<1	<2	<3	<4	<5	5-12	>12	Stars	Triplets
	50	42.72	0	363	29.48	9	22	49	73	120	711	9169	7035	10,1088
4-5	100	39.31	0	283	25.04	10	24	48	62	99	743	9158	7280	94,114
	150	38.46	0	289	24.91	1	14	35	56	100	804	9096	7421	93,912
	200	37.62	0	241	24.95	5	21	43	78	119	896	8985	7497	91,961
4-4.5	50	63.68	0	392	42.63	6	14	30	38	61	295	9644	7567	155,802
	100	60.00	0	425	38.89	2	8	19	28	50	274	9675	7680	146,461
	150	58.21	0	469	38.06	2	11	26	38	54	311	9635	7720	142,174
	200	56.77	0	415	36.66	5	17	30	43	68	336	9596	7744	139,815
	500	55.49	0	357	36.70	8	19	41	55	76	363	9561	7773	136,923

[a]SVM training: svmtrain –s 3 –t 2 –g ga x; SVM estimating: svmpredict tv x.model o
[b]Sampling method: 4-5 sampling FOV=5deg, interval=FOV/2, number of triplet=4; 4-4.5 is similar to 4-5.

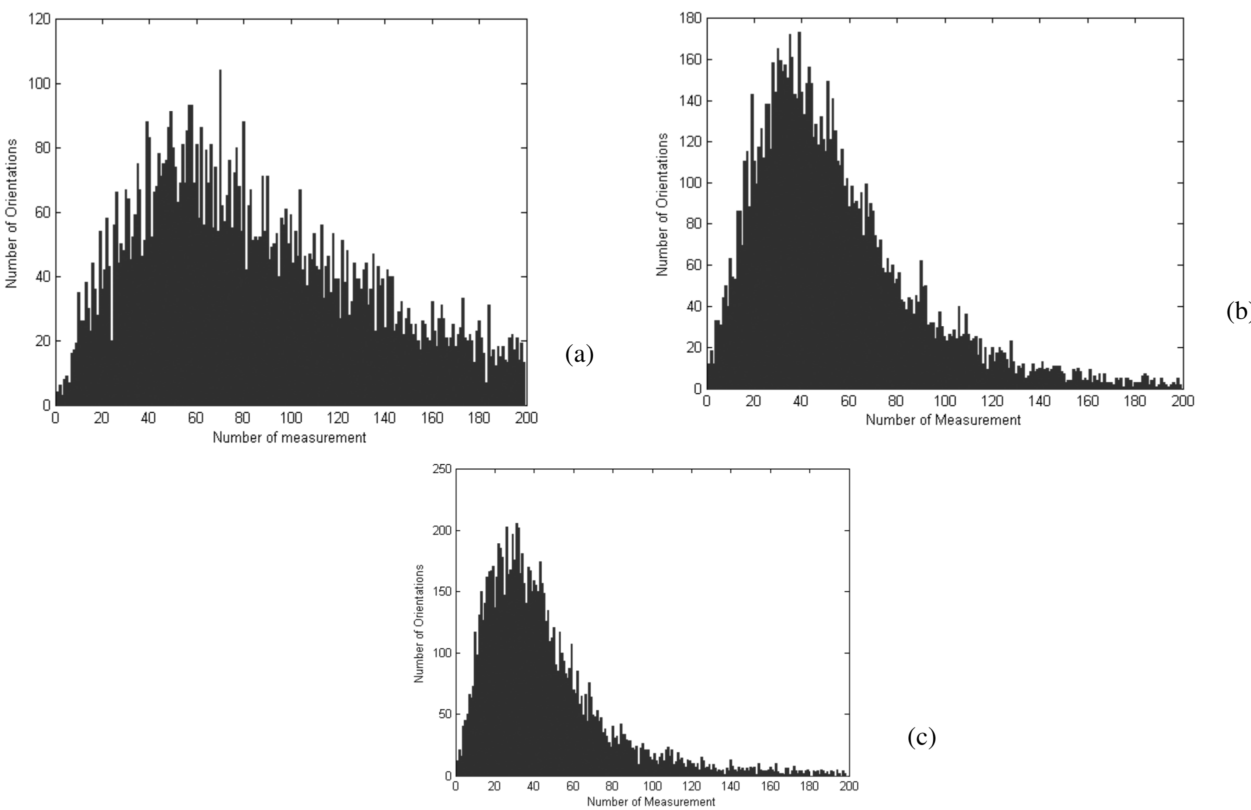

Fig. 4 Comparison of the distribution of the number of triplets measured inside the 8 x 8 deg squared FOV using the different guide star catalog.

Figure 4 shows the comparison of the distribution of the number of star triplets measured inside the 8 x 8 deg squared FOV using the different guide star catalog, tested in 10,000 random boresight directions. Whereas the horizontal axis is the number of triplets measured inside the FOV and the vertical axis is the percent of the orientations. In part a, the distribution of the number of star triplets measured inside the 8 x 8 deg squared FOV is shown using the 287,540 triplets with 7817 stars (Std=100.2). In part b, the distribution of number of star triplets measured inside the 8 x 8 deg squared FOV is shown using the 142,174 triplets with 7720 stars (Std=38.06) obtained by TRSA. In part c, the distribution of the number of star triplets measured inside the 8 x 8 deg squared FOV is shown using the 93,912 triplets with 7421 stars (Std=24.91) obtained by TRSA. It is shown that the triplet database obtained by the TRSA is much better than the original triplet set, the 93,912 triplets sets' Std is almost one fifth of the 287,540 triplet sets and the number of the guide stars needed in the triplet database is also cut down to 7421, almost 400 stars may be eliminated, while the perfection is lost a little, the proportion of the orientations where the number of triplets viewed is less than 2 is only up from 0.12% to 0.15%. Obviously, the guide star triplet set selected by TRSA provided the near normal distribution with the small standard deviation (STD), which is desirable for consistent attitude accuracy.

VII. Conclusion

The generation of the onboard triplet database is significant for the triplet-based SPR algorithm because the performance and reliability of SPR and attitude determination depend closely on the guide star set and the triplet database. By defining the direction and the LVM of the star triplet, a DLVMT model has been introduced in our novel approach. The optimal selection problem of the guide star triplet has been transformed into a high dimensional and nonlinear regression problem to obtain the DLVMT function, which can be solved by the new SVM based on the SLT. The experiment results demonstrate that the triplet catalog obtained by the SVM-based TRSA has a couple of advantages, including fewer total numbers, smaller catalog size, and better distribution uniformity.

References

[1]Accardo, D., and Giancarlo, R., "Brightness-Independent Start-up Routine for Star Trackers," *IEEE Aerospace and Electronic Systems*, Vol. 38, No. 3, 2002, pp. 813-823.

[2]Liebe, C.C., "Star Tracker for Attitude Determination," *IEEE Aerospace and Electronic Systems*, Vol. 10, No. 6, 1995, pp. 10-16.

[3]Padgett, C., and Kreutz-Delgado, K., "A Grid Algorithm for Autonomous Star Identification," *IEEE Aerospace and Electronics Systems*, Vol. 33, No. 1, 1997, pp. 202-213.

[4]Close, D. S., and Padgett, C. W., "Small Field-of-View Star Identification Using Bayesian Decision Theory," *IEEE Aerospace and Electronic Systems*, Vol. 36, No. 3, 2000, pp. 773-783.

[5]Accardo, D., and Rufino, G., "Star Field Feature Characterization for Initial Acquisition by Neural Networks," *Aerospace Conference Proceedings*, IEEE Paper 2319-2330, 2002.

[6]Kim, H.-Y., and Junkins, J. L., "Self-Organizing Guide Star Selection Algorithm for Star Trackers: Thinning Method," *Aerospace Conference Proceedings*, IEEE Paper 2275–2284, 2002.

[7]Tao, L., Zhou, J. L., Zhang, J. P., and Qian, G. H., "A General Method of the Automatic Selection of Guide Star," *Signal Processing Proceedings*, *1998*, Paper 1616 –1619, 1998.

[8]Li-Hong, L., Tao, L., Yongchen, N., and Fu-en, Z., "Improved All-Sky Autonomous Triangle Star-Field Identification Algorithm," Optical Technique, Vol. 26, No. 4, 2000, pp. 372-374.

[9]Cortes, C., and Vapnik, V., "Support-Vector Networks," *Machine Learning*, Vol. 20, 1995, pp.273-297.

[10]Burges, C. J. C., "A Tutorial on Support Vector Machines for Pattern Recognition," *Knowledge Discovery and Data Mining*, Vol. 2, No. 2, 1998, pp.121-167.

[11]Vapnik, V., Golowich, S., and Smola, A., "Support Vector Method for Function Approximation, Regression Estimation, and Signal Processing," edited by M. Mozer, M. Jordan, and T. Petsche, *Advances in Neural Information Processing Systems 9*, MIT Press, Cambridge, MA, 1997, pp. 281-287.

[12]Guyon, J., Weston, S., Barnhill, S,, and Vapnik, V., "Gene Selection for Cancer Classification Using Support Vector Machines," *Machine Learning*, Vol. 46, 2002, pp.389-422.

[13]Mika, S., Rätsch, G., Tsuda, K., and Schölkopf, B., "An Introduction to Kernel-Based Learning Algorithms," *IEEE Neural Networks*, Vol. 12, No. 2, 2001, pp.181-201.

[14]*SAO - Smithsonian Astrophysical Observatory Star Catalog*, http://tdc-www.harvard.edu/software/catalogs/sao.html.

[15]Ma, J., Zhao, Y., and Ahalt, S., OSU SVM Classifier Matlab Toolbox (v. 3.00), http://www.eleceng.ohio-state.edu/~maj/osu_svm/osu_svm3.00.zip.